

Homework 6: Sensors Technologies and Sensor Calibration

Assigned: Mon. 13-March

Due: Wed. 22-March

1. (5 points total) Next to each of the following sensors write the quantity that is measured by the sensor: angular rate, angle, acceleration, direction to North, position, range (distance) to obstacle, bearing to obstacle, orientation, forces, torques. More than one answer may apply.

- RADAR
- GPS
- Accelerometer
- Gyroscope
- LIDAR
- Potentiometer
- Optical Encoder
- Magnetometer
- SONAR
- Infrared
- Strain gauge

2. (5 points total) Starting from the equation $\mathbf{y} = \mathbf{A}\mathbf{p}$ prove that $\mathbf{p} = \mathbf{A}^+\mathbf{y}$ gives the parameter vector \mathbf{p} that minimize the least-square error $J = \|\boldsymbol{\epsilon}\|^2$, where $\boldsymbol{\epsilon} = \mathbf{y} - \mathbf{A}\mathbf{p}$ and $\mathbf{A}^+ = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T$ is the pseudoinverse.

3. (15 points total) An experiment was conducted in which an ultrasonic ranging sensor (Fig. 1a) was used to measure the *two-way travel-time* (TWTT) to an obstacle. The TWTT is the total time it takes a sound waves that is emitted from the sensor to reach the obstacle, reflect and return to the sensor. The distance d between the obstacle and the sensor was varied – beginning from 0.25 m and progressing in increments of 0.25 m to a final distance of 2.50 m. At each fixed distance “station” the obstacle was held stationary for about 10 seconds. The sensor’s output was recorded during the entire process, including during transitions from one station to the next. A log-file containing 1884 measurements of TWTT (in units of micro-seconds) is provided on Blackboard as `data.txt`. You can load this data into MATLAB by placing `data.txt` into your working directory and assigning its contents to a variable `data` using the command `data = load('data.txt')`. Assume that the sensor’s output (the TWTT, denoted t) can be modelled as a function of distance to the obstacle using the following equation:

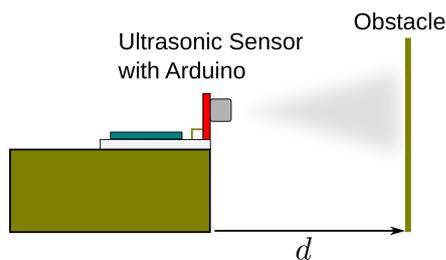
$$t = md + b \tag{1}$$

where d is the distance to the obstacle, m is a term that is proportional to twice the inverse of the speed of sound (to account for the fact that sound must travel twice the distance d). In other words, $m = 2/c$ where c is the actual speed of sound. Last, the constant b is a bias inherent in the sensor.

- (a) (3 points) Plot the raw data with measurement number (the index from 1 to 1884) on the x-axis and the two-way travel time (in microseconds) on the y-axis. Use circular markers connected by lines to display the data (i.e., use the `'o-'` option with the `plot` command). Label the axes using a large legible font (size 16 or greater).



(a) The HC-SR04 ultrasonic sensor.
(<https://www.sparkfun.com/products/13959>)



(b) Experimental setup

- (b) (4 points) By inspecting the data, determine appropriate indices that correspond to each distance station during which the target was held fixed for 10 seconds. Store the data from each station as a separate variable, or within a single structure (whichever you prefer). For each station compute the average of the TWTT. Plot the average TWTT vs. the distance in meters: `distMeters = [0.25:0.25:2.5];`. The plot should have the distance (meters) on the x-axis and the average TWTT (micro-seconds) on the y-axis. Use the same plot style as described in (b). We will now estimate the sensor model parameters m and b using linear regression in MATLAB.
- (c) (2 points) Convert Eq. 1 into matrix form (so that it accounts for all the 10 averages you computed in (c) simultaneously). State this matrix equation in expanded form (i.e., with each matrix element written out explicitly).
- (d) (2 points) Solve the matrix equation from (d) (in a least-squares sense). State the values of m and b you estimated (include units). You must use the technique used in class and cannot use other linear regression software such as Excel or other MATLAB built-in functions.
- (e) (2 points) Using the values you estimated in (d) and the the model Eq. 1 to plot the linear regression superimposed over the data you plotted in (c). Submit only one plot for both (c) and (e) but make sure you use a legend to distinguish the two lines. Since this is not a plot of individual data points, do not use circular markers in the line style (i.e., use the default solid line '-' option with the plot command)
- (f) (2 points) Based on your results in (f) what is the speed of sound determined experimentally in meters/sec? How does this compare to the known speed of sound? Note that the experiment was conducted at room temperature.