

Homework 8: Control

Assigned: Wed. 12-April

Due: Wed. 19-April

- (28 points total) You are tasked with designing a PID controller for the rudder of an unmanned underwater vehicle (UUV). The rudder is steered by a servomotor whose control input is the voltage u . The rudder is modeled as having three states: the current in the motor windings x_1 , the rudder angle x_2 , and the angular rate of the rudder x_3 . The system dynamics are

$$\dot{\mathbf{x}} = \mathbf{f}(t, \mathbf{x}, \mathbf{u})$$

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} (-R/L)x_1 - (K/L)x_3 + (1/L)u \\ x_3 + (W)x_2 \\ (-B/J)x_3 + (K/J)x_1 \end{pmatrix}$$

where the constant parameters are given by

```
% system dynamics parameters
R = 6; % resistance of one motor phase, ohm
L = 10e-3; % inductance of one motor phase, H
B = 2e-5; % motor viscous friction coefficient, N-m-s
J = 2.5e-6; % total mechanical inertia, kg-m^2
K = 3e-2; % electrical constant, V-s/rad
W = -20; % rudder dimensionalized drag coefficient
```

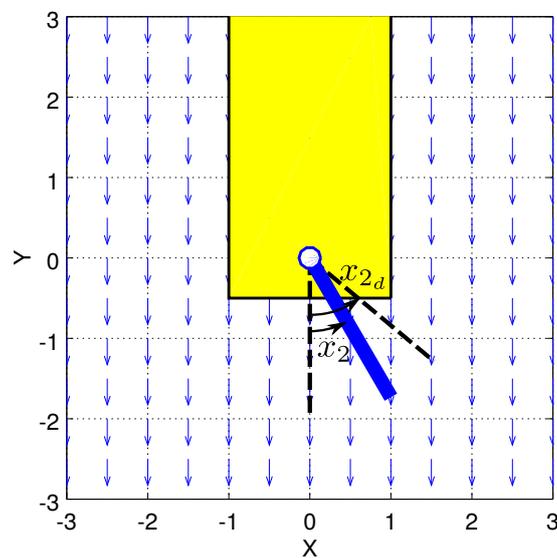


Figure 1: Sketch of a rudder control system for a UUV.

The rudder begins at time $t_0 = 0$ at the state $\mathbf{x}(t_0) = \mathbf{x}_0 = (0 \ 0 \ 0)^T$ (in other words with no current in the motor windings, pointing directly downward, and with zero angular velocity). The desired set-point to which we want to steer the rudder is: $x_{2_d} = 50\pi/180$ (rad). At every time-step in your simulation, you must compute the control input u (i.e., the voltage) to apply to the servomotor and drive the rudder to the desired set-point. Because you are implementing this controller on a digital computer, use a PID-like control law of the form:

$$u(i) = k_p \cdot \text{error_curr}(i) + k_d \cdot \text{error_diff}(i) + k_i \cdot \text{error_cuml}(i);$$

where **error_cur** is the error at the current time-step i , **error_diff** is the difference between the current error (at time step i) and the previous error (at time-step $i - 1$), and **error_cuml** is the summation of all errors from the initial time-step up to time-step i . You may use the function `playRudderResponse.m` to visualize the systems response (but this is optional and is not required to complete this assignment).

- (a) (10 points) Simulate the above system with the following gains $k_p = 15$, $k_d = 200$, and $k_i = 0$ for a period of $T = 0.25$ sec. using a timestep of $h = 0.0001$, and submit one (1) copy of your MATLAB code and one (1) plot of the state history of the x_2 (rudder angle) variable as a function of time. Label your axes and use a large legible font. Include solid and dashed lines on your plot to indicate the setpoint and the values that are $\pm 5\%$ above and below your setpoint as shown in the following figure:

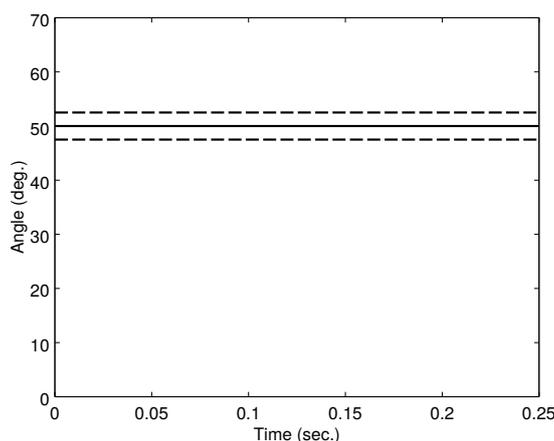


Figure 2: Your plot should include the following dashes and solid lines (in addition to the response $x_2(t)$)

- (b) (10 points) Using a ruler, draw over your plot from Part A and indicate the following on the plot:
- t_r , rise time
 - t_p , peak time
 - t_s , settling time
 - M_p percent overshoot
 - ϵ_{ss} , steady-state error

Also make sure to indicate the numeric value for each of the above variables.

- (c) (2 points) In Part A, what was the maximum and minimum control input (voltage) applied to the servomotor over the course of the simulated time interval?
- (d) (2 points) Set $k_d = k_i = 0$ and iteratively increase the gain in increments of 1 starting from $k_p = 15$ (i.e., modify k_p and re-run your simulation each time). At what value of k_p does the

system become unstable? Note that for the system to be considered unstable the angle must diverge (not just oscillate).

- (e) (6 points) Tune the k_p , k_d and k_i values so that the system has negligible steady state error at 0.15 sec. and the maximum angle of the rudder during the simulation is no greater than 70 deg. State your final k_p , k_d and k_i values. Submit one (1) plot of the corresponding "tuned" state history of the x_2 (rudder angle) variable as a function of time. Label your axes and use a large legible font.